November 6, 2009

MATH 407 MIDTERM EXAM SAMPLE

EXAM OUTLINE

Calculators are not allowed for this exam. The exam will consist of 6 questions. Questions 1-4 are worth 50 points each and questions 5 and 6 are worth 25 points each for a total of 250 points. The content of each question is as follows.

Question 1: (50 points) The first question has two parts. In the first part you will be asked to recite the definition of one or more terms from the class notes and Chapters 1–3 of the text. The precise list of terms that you need to know can be found in the weekly overviews for weeks 1–3. In the second part, you will asked to provide both the statement and the proof (**as given in the on-line notes, not from the book**) of one of the three key theorems in this course: *(1) The Weak Duality Theorem, (2) The Fundamental Theorem of Linear Programming, and (3) The Strong Duality Theorem.*

Question 2: (50 points) In this question you will be asked to model a given problem as an LP. The particular model will be chosen from the models 1 through 18 found on the class webpage.

Question 3: (50 points) In this question you will be asked to solve a two dimensional LP graphically and state the dual of this LP. Please bring a straight-edge and whatever other graphing aids you require.

Question 4: (50 points) In this question you will be asked to solve an LP using the two phase simplex algorithm.

Question 5: (25 points) In this question you will be asked to convert an LP to standard form.

Question 6: (25 points) In this question you will be asked to apply *The Complementary Slackness Theorem* to determine if a given point solves a given LP.

SAMPLE QUESTIONS

1. Part 1:
   i. What does it mean to say that an LP is unbounded? Provide and example of an unbounded LP.
   ii. What is a dictionary for an LP in standard form?
   iii. What is the relationship between dictionaries and simplex tableaus for LPs in standard form?

   Part 2: State and prove the Weak Duality Theorem of Linear Programming for the LP

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & AX \le b,\ 0 \le x\ . \end{array}$$

2. A company needs to lease warehouse storage space over the next 5 months. Just how much space will be required in each of these months is known. However, since these space requirements are quite different, it may be most economical to lease only the amount needed each month on a month-by-month basis. On the other hand, the additional cost for leasing space for additional months is much less than for the first month, so it may be less expensive to lease the maximum amount needed for the entire 5 months. Another option is the intermediate approach of changing the total amount of space leased (by adding a new lease and/or having an old lease expire) at least once but not every month.

   The space requirement (in thousands of square feet) and the leasing costs (in hundreds of dollars) for the various leasing periods are as follows:

Month	Required Space	Leasing Period (months)	Cost ($) per 1,000 square feet leased
1	30	1	650
2	20	2	1000
3	40	3	1,350
4	10	4	1,600
5	50	5	1,900

The objective is to minimize the total leasing cost for meeting the space requirement. Formulate the linear programming model for this problem. You **do not** have to put the LP into standard form.

3. Consider the following LP:

$$\begin{array}{ll} \text{maximize} & 4x - y \\ \text{subject to} & \begin{array}{rcrcr} -2x & + & y & \leq & 4 \\ x & + & y & \leq & 7 \\ 2x & - & y & \leq & 1 \\ x & + & y & \geq & 1 \\ x & \leq & 2, & 0 & \leq \; y \end{array} \end{array}$$

   (a) Solve this LP graphically using the technique described in the class notes. For full credit you will need to able all constraints (along with little arrows indicating the correct side), the feasible region, the objective normal, the solution (with numerical coordinates), and the optimal value.

   (b) State the dual to this LP.

4. Solve the following LP using the simplex algorithm in *t*ableau format (zero credit will be given for solutions using dictionary format). State the solution, the solution to the dual, as well as the associated optimal value.

$$\begin{array}{lrcrcrcl} \text{maximize} & & & 2y & + & 3z & & \\ \\ \text{subject to} & -x & + & y & - & z & \leq & 1 \\ & x & - & 2y & & & \leq & 0 \\ & x & & & + & 3z & \leq & 1 \\ & 0 & \leq & x, & y, & z & & \end{array}$$

5. Put the following LP into standard form:

$$\begin{array}{lrcrcrcr} \text{minimize} & 4x_1 & - & 2x_2 & + & x_3 & & \\ & -x_1 & + & 3x_2 & - & x_3 & \geq & -1 \\ & & & 5x_2 & + & 3x_3 & = & 5 \\ & x_1 & + & x_2 & + & x_3 & \leq & 1 \\ & -1 & \leq & x_2, & -2 & \leq \; x_3 & \leq & 2 \end{array}$$

6. Does the point $x = (0, 1, 1, 1)^T$ solve the following LP?

$$\begin{array}{lrcrcrcrcr} \text{maximize} & 4x_1 & + & 13x_2 & + & 3x_3 & - & 2x_4 & & \\ & x_1 & + & x_2 & + & x_3 & + & x_4 & \leq & 4 \\ \text{subject to} & 2x_1 & + & 3x_2 & + & x_3 & - & x_4 & \leq & 3 \\ & x_1 & - & 2x_2 & - & 2x_3 & + & 3x_4 & \leq & -1 \\ & & & 2x_2 & - & x_3 & - & 2x_4 & \leq & -1 \\ & & & & & 0 \leq x_1, \; x_2, \; x_3, \; x_4 & & & & \end{array}$$