

ERRATA TO “SPECTRUM OF DYNAMICAL SYSTEMS ARISING FROM DELONE SETS”

BORIS SOLOMYAK

1. Lenz and Stollman [3] pointed out that the “metric” ρ defined on page 266 of [4] does not satisfy the triangle inequality. There we used for two Delone sets Λ_1 and Λ_2 , denoting by B_r the ball of radius r centered at the origin:

$$\rho(\Lambda_1, \Lambda_2) = \min\{2^{-1/2}, \tilde{\rho}(\Lambda_1, \Lambda_2)\}, \quad \text{where}$$

$$\tilde{\rho}(\Lambda_1, \Lambda_2) = \inf\{\epsilon > 0 : d_H[\Lambda_1 \cap B_{1/\epsilon}, \Lambda_2 \cap B_{1/\epsilon}] \leq \epsilon\}.$$

Here d_H is the Hausdorff distance. There are several ways to fix this, described in [3]. Alternatively, we can consider

$$\tilde{\varrho}(\Lambda_1, \Lambda_2) = \inf\{\epsilon > 0 : (\Lambda_1 \cap B_{1/\epsilon}) \subset (\Lambda_2 \cap B_{1/\epsilon+\epsilon}) + B_\epsilon; (\Lambda_2 \cap B_{1/\epsilon}) \subset (\Lambda_1 \cap B_{1/\epsilon+\epsilon}) + B_\epsilon\}$$

and use this function instead of $\tilde{\rho}$ above to define a metric. This is essentially equivalent to the topology described in the book Israel [1] on p.154: instead of the Hausdorff distance he just considers a one-to-one correspondence between points of Λ_1 and Λ_2 in $B_{1/\epsilon}$ such that the corresponding points are within ϵ (for ϵ sufficiently small).

We note that the results of [4] are not affected by this error, since we only used the topology and not the precise constants for the metric. We mention also that in the spaces of Delone sets of finite local complexity another metric, ρ' , was used in [4]; the triangle inequality for ρ' is verified in [2, p.1005].

2. In the proof of Theorem 3.1 on pp.270-271, we need uniform continuity rather than continuity to extend the eigenfunction from a dense orbit to the whole space. However, the proof demonstrates exactly the uniform continuity, so there is no problem here.

3. On p.269 it is stated that “being a repetitive Meyer set does not imply... the presence of non-trivial discrete spectral component.” This is not exactly right. In fact, Strungaru [5] proved that such a set always has a relatively dense set of Bragg peaks. It is easy to see that if Λ is a subset of a lattice, then the dual lattice will consist of eigenvalues. In the example given in the paper, $\Lambda \subset \mathbb{Z}$, and the set of eigenvalues coincides with \mathbb{Z} . The discrepancy arises because of the difference in considering the substitution \mathbb{Z} -action and the \mathbb{R} -action on the space of Delone sets.

REFERENCES

- [1] R. Israel, *Convexity in the theory of lattice gases*, Princeton Univ. Press, 1979.
- [2] J.-Y. Lee, R. V. Moody and B. Solomyak, Pure point dynamical and diffraction spectra, *Annales Henri Poincaré* **3** (2002), 1003–1018.
- [3] D. Lenz and P. Stollmann, Delone dynamical systems and associated random operators, Conference Proceedings, Constanta (Romania), July 2-7, 2001, J.-M. Combes, J. Cuntz, G.A. Elliott, G. Nenciu, H. Siedentop, S. Stratila (eds.), Theta Foundation, ISBN 973-85432-2-3. (arXiv:math-ph/0202042)
- [4] B. Solomyak, Spectrum of dynamical systems arising from Delone sets, in *Quasicrystals and Discrete Geometry*, The Fields Institute for Research in Mathematical Sciences Monograph Series **10** (1998), 265–275, American Mathematical Society.
- [5] N. Strungaru, Almost periodic measures and long-range order in Meyer sets, *Discrete Comput. Geom.* **33** (2005), no. 3, 483–505.