Random flag complexes are a natural generalization of random graphs to higher dimensions, and since every simplicial complex is homeomorphic to a flag complex this puts a measure on a wide range of possible topologies. In this talk, I will discuss the recent proof that according to the Erdős–Rényi measure, asymptotically almost all $d$-dimensional flag complexes only have nontrivial (rational) homology in middle degree $\lfloor d/2 \rfloor$. The highlighted technique is originally due to Garland -- what he called "($p$)-adic curvature" in a somewhat different context. This method allows one to prove cohomology-vanishing theorems by showing that certain discrete Laplacians have sufficiently large spectral gap. This reduces certain questions in probabilistic topology to questions about random matrices.