The operations of time shift ($f(t) \rightarrow f(t+1)$) and frequency shift ($f(t) \rightarrow e^{2\pi i\omega t}f(t)$) are fundamental ingredients of applied Fourier analysis, and the group of operators on $L^2(\mathbb{R})$ that they generate gives a unitary representation of the so-called discrete Heisenberg group. How does this representation decompose into irreducible representations? The answer provides illustrations of (i) some useful tools of modern harmonic analysis, when $\omega$ is rational, and (ii) some pathological phenomena from the dark side of representation theory, when $\omega$ is irrational. We shall discuss these results after providing a bit of background on unitary representation theory.