Brownian motion under Knightian uncertainty and path-dependent risk

A typical measure of risk in finance must take into account of the uncertainty of probability model itself (called Knightian uncertainty). Nonlinear expectation and the corresponding non-linear distributions provides a deep and powerful tool: cumulated nonlinear i.i.d random variables of order $1/n$ tend to a maximal distribution, according a new law of large number, whereas, the accumulation of order $1/\sqrt{n}$ tends to a nonlinear normal distribution which is the corresponding central limit theorem. The continuous time uncertainty cumulation forms a nonlinear Brownian motion.

The related stochastic calculus under Knightian uncertainty provides us a powerful tool of valuation for path-dependent derivatives. The corresponding Feynman-Kac formula gives one to one correspondence between fully nonlinear parabolic partial differential equations and backward stochastic differential equations driven by the nonlinear Brownian motion.