One likely model for the shape of the universe is the Poincaré dodecahedral space, which is a quotient of the 3-sphere by the action of the icosahedral group. To help cosmologists, Jeff Weeks adopted a method originally proposed by Klein to find all the spherical harmonics invariant under the icosahedral and other polyhedral groups. In trying to connect the method to polyhedrally-invariant functions on the 2-sphere, we discovered an interesting connection to self-mappings of the 2-sphere, opening the door to a new technique for mathematical art. (Joint work with Jeff Weeks.)