Back in the 1960's, Gian-Carlo Rota recognized the importance of an interpretation for the coefficients in inclusion-exclusion counting formulas as reduced Euler characteristics of associated simplicial complexes. We will begin by discussing some of the history of this powerful link between combinatorics and topology. Then we will turn to the example of weak Bruhat order. This partial order on permutations captures much of the structure of how permutations may be written as products of adjacent transpositions. The relations satisfied by the adjacent transpositions, the braid relations, may be shown to control a poset invariant called the Möbius function -- the aforementioned reduced Euler characteristic in disguise. A topological result of Quillen, the Quillen Fiber Lemma, enables key properties of weak Bruhat order to be transferred to posets of current interest in the representation theory of Kac-Moody algebras, the so-called crystal graphs of highest weight representations. However, computer computations of the Möbius function for crystal graphs led us also to discover unexpected new relations amongst the crystal raising and lowering operators. This work on crystal graphs is joint work with Cristian Lenart.

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