One aspect of complex dynamical systems concerns the study of iterating rational maps on the Riemann sphere. A wealth of complicated and deep behavior can emerge when a rational map is iterated; this behavior is governed by the orbits of the critical points of the map under iteration. A rational map is said to be postcritically finite if every critical point eventually maps into a periodic cycle. Postcritically finite maps are particularly nice to study and are of central importance in complex dynamics. Loosely speaking, in the space of all rational maps, those that are postcritically finite play a role akin to the rational numbers in the reals. In this talk, we explore potential answers to the question in the title that come from complex dynamics, focusing on postcritically finite rational maps. Parts of this talk are joint with X. Buff and A. Epstein, and L. DeMarco and C. McMullen.

Sarah Koch is an Assistant Professor of Mathematics at the University of Michigan. Her research combines topology, algebraic geometry, and complex analysis to understand complex dynamical systems and their associated dynamical moduli spaces. She is an invited plenary speaker at the AMS-MAA Joint Math Meetings in 2019.

Related Links:
Sarah C. Koch
Pacific Institute for the Mathematical Sciences