Combinatorial Gauss-Bonnet Theorem and its applications

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In this talk we will start with the concept of combinatorial curvature on planar graphs. After brief explanation for some progress related to combinatorial curvature, the main topic of this talk will come in, the combinatorial Gauss-Bonnet theorem. Definitely it is the combinatorial counterpart to Gauss-Bonnet theorem in differential geometry. We will especially focus on the Gauss-Bonnet formula involving boundary (left) turns, since we found at least two reasonable applications of it.

The first application is related to the He-Schramm conjecture [1] about types of disk circle packing, which was later proved by Repp [2]. During the talk a statement stronger than the He-Schramm conjecture (i.e., Repp’s theorem) will be presented, and one will see that the stronger version can be proved in a simpler way.

The next application is about isoperimetric constants on planar graphs. Suppose a given planar graph has faces and vertices whose degrees are at least p and q, respectively, where p and q are natural numbers such that 1/p + 1/q < 1/2. Then it is natural to guess that the isoperimetric constant of this graph is at least that of the (p; q)-regular graph, the q-regular planar graph all of whose faces have the same degree p. This ‘guess’ was in fact conjectured by Lawrencenko, Plummer, and Zha [3], for which we could give an affirmative answer using the combinatorial Gauss-Bonnet theorem. A sketch of the proof will be given if time allows.
