PhD Preliminary Exams

The preliminary exams, or "prelims," are the examinations required by the department for admission to official candidacy for the PhD degree. Old prelims are available on-line.

Starting in 2018, the prelims will be administered twice a year: once in September (1st try) and once over the spring break (2nd try). See “Exam Process” section below for changes to the eligibility criteria.

Exam Dates

In 2018, the 1st try of the prelims will be offered September 10-13, 9:30am-1:30pm:

- **Algebra:** Monday, September 10
- **Real Analysis:** Tuesday, September 11
- **Complex Analysis:** Wednesday, September 12
- **Manifolds:** Thursday, September 13

The second try for all of the prelims will take place on Wednesday, March 27, 10:00am-2:00pm.

Exam Process

These exams, four hours in length, are offered every year in the mathematical subjects treated by the designated core courses: algebra, real analysis, complex analysis, and topology and geometry of manifolds.

A student may substitute completion of a full three-quarter sequence of a designated core course, in which grades of 3.8 or above are received each quarter, for the passing of the corresponding preliminary exam. Only one exam can be replaced in this manner.

The written preliminary exams are given twice a year: once during the week in September that precedes the start of Autumn Quarter by two weeks, and once during the break that follows Winter Quarter. Each exam is four hours long, starting at 9:30 a.m. Only the students who have completed the corresponding year-long core sequence with a grade of at least 3.0 each quarter are eligible for the 2nd try.

The exams are written and graded by a committee, which delivers the results to the Graduate Program Committee. The Graduate Program Committee then makes decisions about student performances. All students receive notes describing their performances by the end of the week following the exam.

The prelims are intended to serve as an objective measure of a student's mastery of basic graduate level mathematics.

Normally, students take several exams in September of year two, but students are welcome to attempt exams in September of year one. A student is expected to pass two of the four exams, with at least one pass in either algebra or manifolds, by the September
beginning the student's second year in the Ph.D. program. A student who fails to do so by the end of the second year may be advised to discontinue studies.

Exam Topics

There are four exams:

- **Algebra**: Topics at the level of 402-3-4 and 504-5-6.
- **Real Analysis**: Topics at the level of 424-5-6 and 524-5-6.
- **Complex Analysis**: Topics at the level of 534-5-6.
- **Manifolds**: Topics at the level of 544-5-6.

Each syllabus below lists certain topics that have appeared on the exams. This list is advisory only – it is intended to suggest the level of the exams, not to prescribe exactly the material that will appear. Past exams can be a useful source of practice questions, but a student need not master all material that has been covered on these exams. A student who knows the material in the syllabus and who has spent some time solving problems should do well on the exams.

### Algebra

**Topics:** Linear algebra (canonical forms for matrices, bilinear forms, spectral theorems), commutative rings (PIRs, UFDs, modules over PIDs, prime and maximal ideals, noetherian rings, Hilbert basis theorem), groups (solvability and simplicity, composition series, Sylow theorems, group actions, permutation groups, and linear groups), fields (roots of polynomials, finite and algebraic extensions, algebraic closure, splitting fields and normal extensions, Galois groups and Galois correspondence, solvability of equations).


### Real Analysis

**Topics:** Elementary set theory, elementary general topology, connectedness, compactness, metric spaces, completeness. General measure theory, Lebesgue integral, convergence theorems, $L^p$-spaces, absolute continuity.


### Complex Analysis


**References:** Ahlfors, *Complex Analysis*; Conway, *Functions of One Complex Variable, vol. 1*; Rudin, *Real and Complex Analysis* (the chapters devoted to complex analysis).

### Manifolds

**Topics:** Elementary manifold theory; the fundamental group and covering spaces; submanifolds, the inverse and implicit function theorems, immersions and submersions; the tangent bundle, vector fields and flows, Lie brackets and Lie derivatives, the Frobenius
theorem, tensors, Riemannian metrics, differential forms, Stokes's theorem, the Poincaré lemma, deRham cohomology; elementary properties of Lie groups and Lie algebras, group actions on manifolds, the exponential map.

References: Lee, *Introduction to Topological Manifolds*, 2nd ed. (Chapters 1-12) and *Introduction to Smooth Manifolds*, 2nd ed. (all but Chapters 18 and 22); Massey, *Algebraic Topology: An Introduction* or *A Basic Course in Algebraic Topology* (Chapters 1-5); Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry* (Chapters 1-6); and Warner, *Foundations of Differentiable Manifolds and Lie Groups* (Chapters 1-4).