PhD Preliminary Exams

The preliminary exams, or “prelims,” are the examinations required by the department for admission to official candidacy for the PhD degree. Old prelims are available on-line.

Starting in 2018, the prelims will be administered twice a year: once in September (1st try) and once over the spring break (2nd try). See “Exam Process” section below for changes to the eligibility criteria.

Exam Dates

In 2020, the first try of the prelims will be offered September 14-17, 10am-2pm:

- **Algebra**: Monday, September 14
- **Analysis**: Tuesday, September 15
- **Manifolds**: Thursday, September 17

The second try for all of the prelims will take place during the week of March 22-26, 2021, 10:00am-2:00pm. (Exact dates will be announced in January 2021.)

Exam Process

These exams, four hours in length, are offered every year in the mathematical subjects treated by the designated core courses: algebra, analysis, and topology and geometry of manifolds.

A student may substitute completion of a full three-quarter sequence of a designated core course, in which grades of 3.8 or above are received each quarter, for the passing of the corresponding preliminary exam. Only one exam can be replaced in this manner.

The written preliminary exams are given twice a year: once during the week in September that precedes the start of Autumn Quarter by two weeks, and once during the break that follows Winter Quarter. Each exam is four hours long, starting at 10:00 a.m. Only the students who have completed the corresponding year-long core sequence with a grade of at least 3.0 each quarter are eligible for the 2nd try.

The exams are written and graded by a committee, which delivers the results to the Graduate Program Committee. The Graduate Program Committee then makes decisions about student performances. All students receive notes describing their performances by the end of the week following the exam.

The prelims are intended to serve as an objective measure of a student’s mastery of basic graduate level mathematics.

Normally, students take several exams in September of year two, but students are welcome to attempt exams in September of year one. A student is expected to pass two of the three exams by the September beginning the student’s second year in the Ph.D.
program. A student who fails to do so by the end of the second year may be advised to discontinue studies.

Students’ experiences in the program and on the tests are important to us. If you have a temporary health condition or permanent disability that requires accommodations for exams (conditions include but not limited to mental health, attention-related, learning, vision, hearing, physical or health impacts), you are welcome to contact DRS at 206-543-8924 or uwdrs@uw.edu or disability.uw.edu.

DRS offers resources and coordinates reasonable accommodations for students with disabilities and/or temporary health conditions. Reasonable accommodations are established through an interactive process between you, DRS, and the GPC. If you have already established accommodations with Disability Resources for Students (DRS), please communicate your approved accommodations to the GPC at your earliest convenience.

It is the policy and practice of the University of Washington to create inclusive and accessible learning environments consistent with federal and state law.

Exam Topics

There are three exams:

- **Algebra**: Topics at the level of 402-3-4 and 504-5-6.
- **Analysis**: Topics at the level of 424-5-6, 524-5, and 534.
- **Manifolds**: Topics at the level of 544-5-6.

Each syllabus below lists certain topics that have appeared on the exams. This list is advisory only – it is intended to suggest the level of the exams, not to prescribe exactly the material that will appear. Past exams can be a useful source of practice questions, but a student need not master all material that has been covered on these exams. A student who knows the material in the syllabus and who has spent some time solving problems should do well on the exams.

**Algebra**

**Topics:**

- **Linear algebra**: vector spaces and linear operators, characteristic and minimal polynomials, eigenvalues and eigenvectors, Cayley-Hamilton theorem, Jordan canonical form, rational canonical form.
- **Commutative rings**: PIDs, UFDs, modules over PIDs, prime and maximal ideals, Noetherian and Artinian rings and modules, Hilbert basis theorem, local rings and Nakayama lemma, localization, Integral extensions, Noether normalization lemma, Hilbert Nullstellensatz, prime ideal spectrum.
- **Rings and modules**: simple modules, composition series, Jordan-Holder theorem for modules, semi-simple rings, Artin-Wedderburn theorem, tensor product.
- **Group theory**: nilpotence, solvability and simplicity, composition series, Sylow theorems, group actions, free groups, simple groups, permutation groups, and linear groups, direct and semi-direct product of groups, presentations in terms of generators and relations.
- **Representation theory**: group algebras, irreducible representations, Schur's lemma, Maschke's theorem, character theory.
- **Field theory**: roots of polynomials, finite and algebraic extensions, algebraic closure, splitting fields and normal extensions, finite fields, Galois groups and Galois correspondence, solvability of equations.
- **Category theory and homological algebra**: categories and functors, natural transformations, universal properties, products and coproducts, exact and split exact sequences, 5-lemma and snake lemma, projective and injective modules, resolutions, (left and right) exact functors, adjoint functors, adjointness of Hom and Tensor, Tor and Ext.

Analysis

Topics in Real Analysis: Metric spaces. General measure and general integration theory, Lebesgue integral, convergence theorems. Banach spaces, Hilbert spaces, $L^p$-spaces. Differentiation and its relation to integration in $\mathbb{R}^n$, signed measures, the Radon-Nikodym Theorem, representation of bounded linear functionals on $C_0(X)$ for locally compact Hausdorff spaces $X$.

References: Folland, Real Analysis; Royden, Real Analysis; Rudin, Real and Complex Analysis; Stein and Shakarchi, Real Analysis.

Topics in Complex Analysis: Basic theory of analytic functions from complex numbers to power series to contour integration, Cauchy’s theorem and applications such as the maximum principle, Schwarz Lemma, argument principle, Liouville theorem etc.

References: Ahlfors, Complex Analysis; Conway, Functions of One Complex Variable, vol. 1; Marshall, Complex Analysis; Rudin, Real and Complex Analysis (the chapters devoted to complex analysis).

Manifolds

Topics: Elementary manifold theory; the fundamental group and covering spaces; submanifolds, the inverse and implicit function theorems, immersions and submersions; the tangent bundle, vector fields and flows, Lie brackets and Lie derivatives, the Frobenius theorem, tensors, Riemannian metrics, differential forms, Stokes's theorem, the Poincare lemma, de Rham cohomology; elementary properties of Lie groups and Lie algebras, group actions on manifolds, the exponential map.

References: Lee, Introduction to Topological Manifolds, 2nd ed. (Chapters 1-12) and Introduction to Smooth Manifolds, 2nd ed. (all but Chapters 18 and 22);