Special Offerings

2018-2019 Course Special Offerings

Undergraduate

- Autumn: Math 380: The Art of Problem Solving
- Spring: MATH 380, MATH/STAT 396, MATH 480 A and B

Autumn Quarter: Math 380 Art of Problem Solving

This course is intended to expose the students to the artful side of problem-solving, with numerous examples from various fields across mathematics, including combinatorics, number theory, algebra, geometry, functional analysis and calculus.

We will explore beautiful and surprising discrete structures through classical problems and puzzles, present useful mathematical "tricks" and methods of problem-solving, and teach the students how to construct, write, and present advanced mathematical proofs.

We will closely follow the textbook "The Art and Craft of Problem-Solving" (any edition) by Paul Zeitz. Sample topics include: mathematical induction; the extreme, pigeonhole, and inclusion-exclusion principles; permutations; invariants; graph theory; generating functions and recurrences; Catalan numbers; polynomials and inequalities; partitions; number-theoretic congruences and number-theoretic functions, Diophantine equations, and the Chinese remainder theorem; functional relations; discrete geometry and trigonometry; discrete and continuous probability. Some of the topics overlap with Math 300 but will be covered in more depth.

Student evaluation will be based on class presentations, weekly homework, and a final exam.

Registration requires instructor permission. Students may request permission after spring quarter 2018 grades have been posted. Please send an email to the instructor (julia@math.washington.edu) with the following information: your UW student number, the list of math and CS classes you took at the University of Washington, the grades you got for them (must include spring 2018 grades), and a short paragraph explaining your interest in taking the class.

Spring Quarter: MATH 380, 396, 480

- 380 - Introduction to Discrete Mathematics
- 396 - Finite Markov Chains
- 480A - Algebraic Complexity Theory
- 480B - Cryptography

Math 380 - Introduction to Discrete Mathematics

In how many ways can we pass out $k$ distinct pieces of fruit to $n$ children? In how many ways may $n$ people sit around a round table?
Does every set of six people contain at least three people who all know each other or a set of at least three people none of whom know each other? If you wonder about these and similar questions, then this is a class for you.

The goal of the class is to provide you with an idea of what Discrete Mathematics is about. We will see a sample of topics such as basic counting techniques, the inclusion-exclusion principle, double counting techniques, the pigeonhole principle, induction and recursions, basics of graph theory, and perhaps even some bits of group theory. A big portion of the class will involve you working on solving problems and making guesses/predictions/conjectures.

Tentative textbook: *Combinatorics Through Guided Discovery* by Kenneth P. Bogart

**Prerequisites:** minimum 2.0 in Math 300.

Students are not eligible for this course if they have previously received credit for Math 380 - The Art of Problem Solving, Math 380 in Winter 2017, Spring 2017, Winter 2018 or Spring 2018 and/or MATH 461/462.

See Spring Time Schedule Notes for registration information.

**Math/Stat 396 - Finite Markov Chains**

Finite Markov chains; stationary distributions; time reversals; classification of states; classical Markov chains; convergence in total variation distance and $L_2$; spectral analysis; relaxation time; Monte Carlo techniques: Rejection sampling, Metropolis-Hastings, Gibbs sampler, Glauber dynamics, Hill climb and Simulated annealing; harmonic functions and martingales for Markov chains.

Students should have earned a minimum grade of 2.0 in Math 308 in addition to listed course prerequisites.

Prerequisite: minimum grade of 2.0 in either MATH 395 or STAT 395, or minimum grade of 2.0 in STAT 340 and in STAT 341.

See Spring Time Schedule Notes for registration information.

**Math 480A - Algebraic Complexity Theory**

The topic of computational complexity is at the heart of theoretical computer science. The famous million dollar millennium problem "P vs NP" asks whether the class of problems that can be solved in polynomial time is the same as the class of problems that can be verified in polynomial time. The course will begin with an introduction to classical complexity theory with a discussion of computability, Turing machines, complexity classes, NP completeness, the Cook-Levin Theorem and the P vs NP problem.

The main goal of the course is to formulate an algebraic computational model where polynomials are computed via arithmetic circuits. We will discuss algebraic analogues of the classes P and NP called VP and VNP, introduced by Valiant in the 70s. The "VP vs VNP" question asks whether the classes VP and VNP are equal, and can be viewed as a simplified version of the P vs NP question. It is hoped that sophisticated techniques in algebra, geometry, and representation theory may be employed to address the VP vs VNP question, which in turn may shed light on the P vs NP question.

**Prerequisites:** Math 300, 308.

**480 B Cryptography**

Much of modern cryptography is based on mathematics. For instance, the security of encryption schemes is based on the hardness of mathematical problems like factoring or the discrete logarithm problem. Additionally, some of the best-known attacks use ideas that underlie the cryptographic protocols. Topics will include attacks on the discrete logarithm problem for black box groups (e.g. Pollard
Prerequisites: Math 300, 308.

Graduate

- Math 516 A: Convex Analysis and Optimization
- Math 581 A: Topics in Geometric Analysis I
- Math 581 C: Geometric Function Theory
- Math 581 D: Algebraic Combinatorics on Simplicial Complexes
- Math 581 E: Microlocal Analysis and Integral Geometry
- Math 582 B: Topics in Number Theory
- Math 582 C: Quasiconformal Geometry
- Math 582 D: Probabilistic Combinatorics
- Math 582 E: Microlocal Analysis and Integral Geometry
- Math 582 H: Hopf Algebras
- Math 583 A: Topics in Geometric Analysis II
- Math 583 D: Combinatorics of Matrix Varieties
- Math 583 E: Theory of Linear and Nonlinear Second Order Elliptic Equations
- Math 583 H: Cohomology Theories, Triangulated Categories and Applications
- Math 600 C: Geometric Inverse Problems

516 A: Drusvyatskiy - Convex Analysis and Optimization

A systematic study of convex sets and functions began at the onset of the 20th century, though many underlying principles date back to antiquity. The development of the subject dramatically accelerated at the end of the last century, when it became clear that (1) convex functions are widespread in applications, and (2) convexity is essential for establishing efficiency guarantees for optimization methods. The “big data” revolution of the last decade has dramatically accelerated the development of the subject still. This course will begin with some classical topics such as continuity and differentiability of convex functions, duality, smoothing, and monotone operators. The algorithmic portion will revolve around computational complexity, first-order methods, and stochastic algorithms. Time permitting, we will also discuss average case guarantees of algorithms for nonconvex problems under natural statistical models on the data generating mechanism.

581 A: Toro - Topics in Geometric Analysis I

Geometric Analysis has seen fundamental breakthroughs in the last few years. The work of Cheeger Naber and Valtorta has given crucial information about the size of the critical set of harmonic functions, and about the the structure and size of the singular set to solutions of variational problems. The progress on these questions is due to the introduction of amazingly simple and powerful tools. Namely:

- Quantitative Differentiation (Cheeger, Naber, Valtorta)
- Quantitative Stratifi (Cheeger, Naber, Valtorta)
The goal of this class is to present these tools and describe some of their applications. The two quarters of *Topics in Geometric Analysis* are completely independent.

**References:**

- *Stratification for the singular set of approximate harmonic maps*, A Naber, D Valtorta (arXiv:1611.03008)
- *Quantitative Reifenberg theorem for measures* N. Edelen, A Naber, D Valtorta (arXiv:1612.08052)

**Prerequisites for Math 581: Real Analysis, basic knowledge of harmonic functions.**

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**581 C: Marshall - Geometric Function Theory**

A fourth quarter in analysis of one complex variable. Topics include the Koebe distortion theorems, harmonic measure and the Beurling projection theorem, capacity and potential theory, extremal length, analysis on finitely connection regions.

Note: this class will cover (in addition to other topics) material prerequisite to the course “Quasiconformal Geometry” proposed by Steffen Rohde.

**Prerequisites: Math 534-5.**

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**581 D: Novik - Algebraic Combinatorics on Simplicial Complexes**

In this class we will concentrate on studying combinatorics of simplicial complexes and their face numbers as well as other combinatorial and algebraic invariants. A simplicial complex is a family of sets closed under inclusion. For instance, the collection of cliques of a given graph is such a complex, and so is the collection of independent subsets of a given set of elements in a vector space. Described geometrically, a simplicial complex consists of vertices, edges, triangles, and higher dimensional simplices “nicely” glued together. Thus another big class of examples comes from triangulations of spheres and, more generally, triangulations of manifolds and pseudomanifolds.

Face vector is a basic combinatorial invariant of a simplicial complex that encodes the number of faces of the complex in each dimension. A typical combinatorial problem is to characterize those integer vectors that can arise as the face vectors of a given family of simplicial complexes. (For instance, is there a graph on 100 vertices that has 1000 edges, 1000 cliques of size three, 2000 cliques of size 4, and no larger sized cliques?) This problem, while very easy to state, turns out to be notoriously difficult, and is still wide open for many interesting classes of complexes. In most cases where solutions (or partial solutions) are known they involve a fascinating interplay of techniques from combinatorics, commutative algebra, and elementary algebraic topology.

In this course we will introduce several such tools and techniques, the most prominent of which is the theory of Stanley-Reisner rings.

**Prerequisites: familiarity with basic commutative algebra notions (such as polynomial rings and ideals) as well as with the basics of algebraic topology (e.g., simplicial homology and the fundamental group) will be assumed, although some of these
notions will be reviewed in class.

581 E: Uhlmann - Microlocal Analysis and Integral Geometry

Microlocal Analysis has proven to be a very useful technique in many fields. See for instance the proof of stability of Kerr de Sitter black holes by Peter Hintz and Andras Vasy. In inverse problems it has had many applications including the solution by Stefanov, Uhlmann and Vasy of the boundary rigidity problem and lens rigidity problem in many cases. It is particularly useful in recovering singularities of medium parameters from boundary or scattering data.

In this course we will introduce the basic ideas of microlocal analysis including wave front set, conormal and Lagrangian distributions and their corresponding calculus. Applications will be given to integral geometry including the X-ray and Radon transform, the attenuated X-ray transform, teh Doppler transform, Tensor tomography, the geodesic X-ray transform among others.

582 B: Greenberg - Topics in Number Theory

This course is intended to be a sequel to Bianca Viray's course in algebraic number theory (Fall, 2017). We will discuss a variety of topics in algebraic number theory. One topic will be studying more deeply the question of how primes decompose in the ring of integers of an algebraic number field $K$. One special case is when $K$ is generated over $\mathbb{Q}$ by roots of unity, the so-called cyclotomic extensions of $\mathbb{Q}$. We will also discuss the Dedekind zeta function attached to $K$, which is a natural generalization of the Riemann zeta function, and Dirichlet $L$-functions. One objective of the course is to present a proof of Dirichlet's famous theorem on primes in arithmetic progressions. It states that if $m \geq 1$ and $a$ is an integer relatively prime to $m$, then there exist infinitely many primes $p$ such that $p \equiv a \pmod{m}$. The proof uses the above mentioned topics. There is a generalization called the Chebotarev Density Theorem which we will also discuss. That topic leads to the study of Artin $L$-functions. All of these topics play a fundamental role in modern number theory.

582 C: Rohde - Quasiconformal Geometry

Quasiconformal homeomorphisms arise naturally throughout different areas of analysis and geometry, for instance as non-smooth limits of diffeomorphisms, and as homeomorphic solutions to the Beltrami differential equation. They are generalizations of conformal maps and first arose in connection with Teichmüller theory and later complex dynamics and hyperbolic geometry. They are flexible enough to allow for generalizations to maps between arbitrary metric spaces, yet rigid enough to allow for a rich regularity theory.

This course provides an introduction to the theory of quasiconformal maps. Topics discussed include:

- The geometric definition via the modulus of curve families
- The analytic definition via the Beltrami equation and its solutions
- Quasisymmetric maps and generalizations to metric spaces -Applications to dynamics, geometry and an introduction to Teichmüller theory

582 D: Rothvoss - Probabilistic Combinatorics

One of the main techniques in combinatorics is the probabilistic method pioneered among others by Paul Erdős. For example if one wanted to show that a certain discrete object indeed exists — say a graph on $n$ vertices with no cliques or independent sets larger than $O(\log n)$ — then one can consider a suitable probability distribution and argue that the desired object occurs with positive probability. In other applications one aims to prove some relation or inequality, say between the number of edges in a graph and the
number of edge crossings in a planar drawing, and the inequality can be derived by estimating expectations of a random experiment.

In this graduate topics course we will cover some of the most beautiful and striking examples from probabilistic combinatorics. Some of the covered material can also be found in the textbook of Alon and Spencer, other more recent developments comprise the method of Dependent Random Choice and the Kuperberg-Lovett-Peled Theorem.

Content
- Chapter 1: The basic probabilistic method
  - Splitting graphs
  - Sum Free Sets
  - Unbalancing lights
  - Graphs with high girth and high chromatic number
  - Brégman's Theorem
  - Concentration: Chernov vs. Martingales
  - The Roedl Nibble

- Chapter 2: The Lovasz Local Lemma
  - The original proof of the LLL
  - Coloring Hypergraphs and Property B
  - The Algorithmic proof by Moser & Tardos

- Chapter 3: Graphs and Planes
  - Motivating example: Point line incidences
  - Bipartite graphs without a $K_{s,s}$ subgraph
  - The Crossing Number Theorem
  - Point line incidences
  - Number of Unit Distances

- Chapter 4: Existence of Rigid Structures — The Kuperberg-Lovett-Peled Theorem
  - Motivating example: $t$-wise independent random strings
  - Proof of the KLP-Theorem
  - Application to $t$-wise independent random strings

- Chapter 5: VC dimension
  - Def. VC Dimension
  - The $\varepsilon$-net Theorem

- Chapter 6: Dependent random choice

- Chapter 7: The Szemerédi Regularity Lemma and applications
  - The Szemerédi Regularity Lemma
  - Testing Triangle Freeness

- Chapter 8: Discrepancy Theory
582 E: Uhlmann - Microlocal Analysis and Integral Geometry

Microlocal Analysis has proven to be a very useful technique in many fields. See for instance the proof of stability of Kerr de Sitter black holes by Peter Hintz and Andras Vasy. In inverse problems it has had many applications including the solution by Stefanov, Uhlmann and Vasy of the boundary rigidity problem and lens rigidity problem in many cases. It is particularly useful in recovering singularities of medium parameters from boundary or scattering data.

In this course we will introduce the basic ideas of microlocal analysis including wave front set, conormal and Lagrangian distributions and their corresponding calculus. Applications will be given to integral geometry including the X-ray and Radon transform, the attenuated X-ray transform, the Doppler transform, Tensor tomography, the geodesic X-ray transform among others.

582 H: Zhang - Hopf Algebras

This course is an introduction to Hopf algebra. In addition to basic material in Hopf algebra, we will present some latest developments in quantum groups and tensor and fusion categories. One of the newer topics is homological properties of noetherain Hopf algebras of low Gelfand-Kirillov dimension. A good reference for the first two topics in the book *Hopf Algebras and Their Action Rings* by Susan Montgomery. Here is a list of possible topics:

1. Classical theorems concerning finite dimensional Hopf algebras.
2. Infinite dimensional Hopf algebras and quantum groups.
3. Duality and Calabi-Yau property.
4. Actions of Hopf algebras and invariant theory.
5. Representations of Hopf algebras, tensor and fusion categories.

There will be three or four homework assignments and students are encouraged to do homework.

Prerequisites. Math 502/3/4 (and second-year graduate algebra is also helpful).

583 A: Becker-Kahn - Topics in Geometric Analysis II

Geometric Analysis has seen fundamental breakthroughs in the last few years. The work of Logunov and Malinnikova has given remarkable information about the size of the nodal sets for the eigenfunctions of the Laplacian, yielding solutions to long standing conjectures.

The progress on these questions is due to the introduction of amazingly simple and powerful tools. Namely:

- Geometric-combinatorial methods introduced to study elliptic eigenvalue problems (Logunov, Malinnikova)

The goal of this class is to present these tools and describe some of their applications. The two quarters of *Topics in Geometric Analysis* are completely independent.

References:

- *Ratios of harmonic functions with the same zero set*, A. Logunov, E. Malinnikova (arXiv:1506.08041)
• Nodal sets of Laplace eigenfunctions: polynomial upper estimates of the Hausdorff measure, A. Logunov (arXiv:1605.02587)
• Nodal sets of Laplace eigenfunctions: proof of Nadirashvili's conjecture and of the lower bound in Yau's conjecture, A. Logunov (arXiv:1605.02589)
• Nodal sets of Laplace eigenfunctions: estimates of the Hausdorff measure in dimension two and three, A. Logunov, E. Malinnikova (arXiv:1605.02595)
• Quantitative propagation of smallness for solutions of elliptic equations, A. Logunov, E. Malinnikova (arXiv:1711.10076)

Prerequisites: Real Analysis, basic knowledge of harmonic functions.

583 D: Billey - Combinatorics of Matrix Varieties
Matrices are fundamental objects in mathematics. In this class, we will describe some families of affine and projective varieties defined by sets of matrices with nice properties. Many geometrical properties of these varieties can be deduced from the combinatorial data in the indexing set. The main examples include determinantal and permanental varieties, Schubert varieties in different contexts, and alternating sign matrix varieties. We will give a very concrete introduction to these varieties, and discuss combinatorial aspects Schubert calculus, cohomology rings, and the structure of their tangent spaces. We will also discuss some of the main open problems in the field. This is an active area of research so students will be asked to give a presentation on a recently published paper to be chosen together with the instructor.

Prerequisites: strong background in algebra and combinatorics at the graduate level equivalent to 502,503,504 and 561,562,563.

583 E: Yuan - Theory of Linear and Nonlinear Second Order Elliptic Equations
Linear Theory: Solvability, a priori estimates, Schauder and Calderon-Zygmund estimates, and regularity

Nonlinear Theory: De Giorgi Nash Moser theory for divergence equations (eg. minimal surface equation), Krylov-Safonov theory for nondivergence equations (eg. Monge-Ampere equation, special Lagrangian equations, Bellman equations, and Isaacs equations).

Contents: Harmonic functions (properties), Schauder for $D_*$, Weighted norm, solvability for Laplace, Boundary Schauder, $L^p$ for $D_*$, Energy method, capacity, Poincare, Sobolev, $W^{1,2}$ or $H^1$ space, trace; De Giorgi/Nash, Harnack, Quick applications of Harnack, Minimal surface equations, Viscosity solutions to Nondivergence equations, Alexandrov maximum principle, Krylov-Safonov, Uniqueness and Existence of viscosity solutions, $C^1,\alpha$ regularity, $C^2,\alpha$ regularity for convex equations, Monge-Ampere and special Lagrangian equations, Bellman equations, and Isaacs equations.

References:
• Lecture notes will be provided.

Prerequisites: Advanced calculus/Real Analysis (Comparable to Rudin's Principles of Mathematical Analysis; Previous PDE knowledge is welcome, but not required)
583 H: Zhang - Cohomology Theories, Triangulated Categories and Applications

This covers some (co)homology theories and structures of triangulated categories recently used in noncommutative algebra and noncommutative algebraic geometry. In addition, we will discuss some latest developments and applications in representation theory and noncommutative algebraic geometry. Some basics (see first two topics) can be found in the book *An Introduction to Homological Algebra* by Charles Weibel. Here is a list of possible topics:

1. Hochschild cohomology and group cohomology.
2. Homotopy categories and derived categories.
3. Dualizing complexes over noncommutative algebras and applications.
5. Semiorthogonal decomposition of derived categories.

There will be three or four homework assignments and students are encouraged to do homework.

**Prerequisites.** Math 502/3/4 (and second-year graduate algebra is also helpful). Students are supposed to know basics such as projective, injective, flat modules, resolutions and derived functions.

600 C: Paternain/Uhlmann - Geometric Inverse Problems

The study of geometric inverse problems is typically motivated by inverse problems in PDEs, geophysics and medical imaging. The main goal is the reconstruction of geometric structures (metrics, connections, vector bundles etc.) from either boundary measurements or local measurements. The course will describe recent developments in the area with an emphasis on the 2D picture.

The first part of the course will include a thorough discussion of the geodesic X-ray transform for functions and tensors when the background manifold is simple (i.e. it has strictly convex boundary, no conjugate points and is simply connected). Then we shall move on to attenuated X-ray transforms, including versions for connections and Higgs fields (systems).

The second part of the course will address non-linear geometric inverse problems. A full proof of boundary rigidity for simple surfaces will be given along with proofs for the recovery of a connection and a Higgs field from scattering data.

While a geometric background will be desirable, it is not strictly necessary, as the relevant tools in 2D can be provided on demand.

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