Special Offerings

2018-2019 Course Special Offerings

Undergraduate

- Autumn: Math 380: The Art of Problem Solving
- Spring:
  - MATH 380
  - MATH/STAT 396
  - MATH 480 A and B

Autumn Quarter: Math 380 Art of Problem Solving

This course is intended to expose the students to the artful side of problem-solving, with numerous examples from various fields across mathematics, including combinatorics, number theory, algebra, geometry, functional analysis and calculus.

We will explore beautiful and surprising discrete structures through classical problems and puzzles, present useful mathematical "tricks" and methods of problem-solving, and teach the students how to construct, write, and present advanced mathematical proofs. We will closely follow the textbook "The Art and Craft of Problem-Solving" (any edition) by Paul Zeitz. Sample topics include: mathematical induction; the extreme, pigeonhole, and inclusion-exclusion principles; permutations; invariants; graph theory; generating functions and recurrences; Catalan numbers; polynomials and inequalities; partitions; number-theoretic congruences and number-theoretic functions, Diophantine equations, and the Chinese remainder theorem; functional relations; discrete geometry and trigonometry; discrete and continuous probability. Some of the topics overlap with Math 300 but will be covered in more depth.

Student evaluation will be based on class presentations, weekly homework, and a final exam.

Registration requires instructor permission. Students may request permission after spring quarter 2018 grades have been posted. Please send an email to the instructor (julia@math.washington.edu) with the following information: your UW student number, the list of math and CS classes you took at the University of Washington, the grades you got for them (must include spring 2018 grades), and a short paragraph explaining your interest in taking the class.

Spring Quarter: MATH 380, 396, 480

- 380 - Introduction to Discrete Mathematics
- 396 - Finite Markov Chains
- 480A - Algebraic Complexity Theory
- 480B - Cryptography

Math 380 - Introduction to Discrete Mathematics

In how many ways can we pass out \( k \) distinct pieces of fruit to \( n \) children? In how many ways may \( n \) people sit around a round table?
Does every set of six people contain at least three people who all know each other or a set of at least three people none of whom know each other? If you wonder about these and similar questions, then this is a class for you.

The goal of the class is to provide you with an idea of what Discrete Mathematics is about. We will see a sample of topics such as basic counting techniques, the inclusion-exclusion principle, double counting techniques, the pigeonhole principle, induction and recursions, basics of graph theory, and perhaps even some bits of group theory. A big portion of the class will involve you working on solving problems and making guesses/predictions/conjectures.

Tentative textbook: *Combinatorics Through Guided Discovery* by Kenneth P. Bogart

**Prerequisites:** minimum 2.0 in Math 300.

*Students are not eligible for this course if they have previously received credit for Math 380 - The Art of Problem Solving, Math 380 in Winter 2017, Spring 2017, Winter 2018 or Spring 2018 and/or MATH 461/462.*

**See Spring Time Schedule Notes for registration information.**

**Math/Stat 396 - Finite Markov Chains**

Finite Markov chains; stationary distributions; time reversals; classification of states; classical Markov chains; convergence in total variation distance and L2; spectral analysis; relaxation time; Monte Carlo techniques: Rejection sampling, Metropolis-Hastings, Gibbs sampler, Glauber dynamics, Hill climb and Simulated annealing; harmonic functions and martingales for Markov chains.

Students should have earned a minimum grade of 2.0 in Math 308 in addition to listed course prerequisites.

Prerequisite: minimum grade of 2.0 in either MATH 395 or STAT 395, or minimum grade of 2.0 in STAT 340 and in STAT 341.

**See Spring Time Schedule Notes for registration information.**

**Math 480A - Algebraic Complexity Theory**

The topic of computational complexity is at the heart of theoretical computer science. The famous million dollar millennium problem "P vs NP" asks whether the class of problems that can be solved in polynomial time is the same as the class of problems that can be verified in polynomial time. The course will begin with an introduction to classical complexity theory with a discussion of computability, Turing machines, complexity classes, NP completeness, the Cook-Levin Theorem and the P vs NP problem.

The main goal of the course is to formulate an algebraic computational model where polynomials are computed via arithmetic circuits. We will discuss algebraic analogues of the classes P and NP called VP and VNP, introduced by Valiant in the 70s. The “VP vs VNP” question asks whether the classes VP and VNP are equal, and can be viewed as a simplified version of the P vs NP question. It is hoped that sophisticated techniques in algebra, geometry, and representation theory may be employed to address the VP vs VNP question, which in turn may shed light on the P vs NP question.

**Prerequisites: Math 300, 308.**

**480 B Cryptography**

Much of modern cryptography is based on mathematics. For instance, the security of encryption schemes is based on the hardness of mathematical problems like factoring or the discrete logarithm problem. Additionally, some of the best-known attacks use ideas that underlie the cryptographic protocols. Topics will include attacks on the discrete logarithm problem for black box groups (e.g. Pollard
rho), index calculus attacks, elliptic curve cryptography, and lattice based cryptography.

Prerequisites: Math 300, 308.

Graduate Special Topics 2019-2020

- Math 516 A: Convex Analysis and Optimization
- Math 581 A: Theory of Linear and Nonlinear Second Order Elliptic Equations
- Math 581 B: Algebraic Groups I
- Math 581 C: Optimal Transport I
- Math 581 E: Data Science
- Math 582 A: Medical Imaging
- Math 582 B: Algebraic Groups II
- Math 582 C: Optimal Transport II
- Math 582 E: Rational Points I
- Math 582 H: Complex
- Math 583 B: Intersection Theory
- Math 583 C: Optimal Transport III
- Math 583 E: Communicating Math
- Math 583 H:
- Math 581 I: Rational Points II

516 A: Drusvyatskiy - Convex Analysis and Optimization

This is an introductory course in convex analysis and optimization, with a focus on applications in data science. Some representative topics include duality, monotone operators, computational complexity, acceleration, splitting methods, stochastic algorithms, and elements of nonsmooth and nonconvex optimization. This course is appropriate for anyone with a working knowledge of linear algebra and mathematical analysis.

Though the course "MATH 581: High dimensional probability for data science (Autumn 2019)" is not a prerequisite, it would help to contextualize the material.

581 A: Yuan - Theory of Linear and Nonlinear Second Order Elliptic Equations Part II

Linear Theory: Solvability, a priori estimates, Schauder and Calderon-Zygmund estimates, and regularity.  
Nonlinear Theory: De Giorgi Nash Moser theory for divergence equations (eg. minimal surface equation), Krylov-Safonov theory for nondonvergence equations (eg. Monge-Ampere equation, special Lagrangian equations, Bellman equations, and Isaacs equations).

Contents: Harmonic functions (properties), Schauder for $\Delta$, Weighted norm, solvability for Laplace, Boundary Schauder, Lp for $\Delta$; Energy method, capacity, Poincare, Soblev, $W^{1,2}$ or $H¹$ space, trace; De Giorgi/Nash, Harnack, Quick applications of Harnack, Minimal surface equations, Viscosity solutions to Nondivergence equations, Alexandrov maximum principle, Krylov-Safonov, Uniqueness and Existence of viscosity solutions, $C^1$ regularity, $C^\infty$ regularity for convex equations, Monge-Ampere and special Lagrangian equations, Bellman equations, and Isaacs equations.

References:
581 B: Alper - Algebraic Groups I

Course description. The topic of algebraic groups is a rich subject combining both group-theoretic and algebro-geometric-theoretic techniques. Examples include the general linear group $\text{GL}_N$, the special orthogonal group $\text{SO}_N$ or the symplectic group $\text{Sp}_N$. Algebraic groups play an important role in algebraic geometry, representation theory and number theory.

In this course, we will take the functorial approach to the study of linear algebraic groups (more generally, affine group schemes) equivalent to the study of Hopf algebras. The classical view of an algebraic group as a variety will come up as a special case of a smooth algebraic group scheme. Our algebraic approach will be independent (even complementary) to the analytic approach taken in the course on Lie groups.

First quarter: Algebraic Groups I, Jarod Alper

We will use the language of schemes following the excellent introductory texts of W. Waterhouse and J.S. Milne. We will also reference the classic texts by A. Borel, J. Humphreys and T.A. Springer.

Topics at a glance:
- Group schemes over an arbitrary base
- Affine group schemes vs Hopf algebras;
- Representations: modules vs. comodules;
- Examples and special cases: abelian group schemes and Cartier duality, etale group schemes, matrix groups, groups of multiplicative type (tori), unipotent groups, nilpotent and solvable groups (i.e. Borel subgroups of semisimple groups);
- Barsotti-Chevalley Theorem on Algebraic Groups;
- Existence of quotients of algebraic groups;
- Detour on descent and algebraic spaces;
- Actions of algebraic groups on schemes and G-torsors;
- Geometric properties: connectedness, irreducibility, smoothness;
- Jordan decompositions;
- Tannaka duality

References:

We will use the language of schemes following the excellent introductory texts of W. Waterhouse and J.S. Milne. We will also reference the classic texts by A. Borel, J. Humphreys and T.A. Springer.
**Prerequisites.** Modern Algebra 504/5/6. The course will be suitable for a 2nd year or above graduate student leaning towards an algebra-related field (understood broadly: combinatorics, representation theory, algebraic geometry, algebraic topology). The second year graduate algebra course “Algebraic structures” is desirable but not required.

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**581 C: Pal - Optimal Transport I**

The modern theory of Monge-Kantorovich optimal transport is barely three decades old. Already it has established itself as one of the most happening areas in mathematics. It lies at the intersection of analysis, geometry, and probability with numerous applications to physics, economics, and serious machine learning.

This year-long graduate topics course will serve as an introduction to this rich and useful theory.

The textbooks will be [1] and [2]. This will be supplemented often with papers, especially on the very recent approach by the so-called Schrodinger problem. We will roughly follow the following outline.

**Fall:** State the transport problem with a general cost function. Analytic description of solutions. Duality. Displacement convexity. Special examples.

**Winter:** The geometry of Wasserstein space. Benamou-Brenier formulation. Otto calculus. Gradient flow of entropy for the heat equation.

**Spring:** Schrodinger problem. Particle systems and the probabilistic view of optimal transportation. Gradient flow of entropy for Fokker-Planck equations.

**References**


Prerequisites: Measure theory and some topology and functional analysis. Basically the graduate real analysis sequence. It is useful to have some knowledge of probability (large deviations, Brownian motion) and Riemannian manifolds, but not necessary, since we will develop some of these notions on the way.

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**581 E: Drusvyatskiy - High Dimensional Probability for Data Science**

This is an introductory course in high-dimensional probability with a view towards applications in data science. The main focus will be on the concentration phenomenon in high dimensions. In parallel, we will use the developed techniques to analyze algorithms for various statistical inverse problems, such as community detection, low-rank matrix completion, phase retrieval, robust principal component analysis, etc. This course is appropriate for anyone with a working knowledge of linear algebra and mathematical analysis.

Textbook: Roman Vershynin, "High-dimensional probability: An introduction with applications in data science."

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**582 A: Uhlmann - Introduction to the Mathematics of Medical Imagining**

In the first part of the course we will study in detail the mathematics of several medical imaging technique like CT (computerized tomography), PET (positron emission tomography), SPECT (single positron emission tomography) and MRI (magnetic resonance imaging). These techniques revolutionized medicine and the development of CT, PET and MRI were awarded the Nobel Prize in...
Physiology. The underlying mathematical objects that are used in these imaging techniques are the Radon transform and the Fourier transform. The Fourier transform and the Radon transform will be studied in detail. Both of them have applications in several other areas besides medical imaging.

In the second part of the course we will study the mathematics of a different medical imaging technique: Optical Tomography. In this case it is measured the response of the body to light in order to create an image of the optical properties of the medium. This problem can be recast as determining the coefficients of a partial differential equation from some properties of the solutions measured at the boundary of some domain. No background is necessary on partial differential equations. What we will need will be developed in the course.

References:

1. C. Epstein, An Introduction to the Mathematics of Medical Imaging.
2. F. Natterer: The Mathematics of Computerized Tomography
3. S. Deans, The Radon transform and some of its applications.

Prerequisites: The Real Analysis Course or its equivalent.

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582 B: Pevtsova - Algebraic Groups II

Course description. The topic of algebraic groups is a rich subject combining both group-theoretic and algebro-geometric-theoretic techniques. Examples include the general linear group GLN, the special orthogonal group SON or the symplectic group SpN. Algebraic groups play an important role in algebraic geometry, representation theory and number theory.

In this course, we will take the functorial approach to the study of linear algebraic groups (more generally, affine group schemes) equivalent to the study of Hopf algebras. The classical view of an algebraic group as a variety will come up as a special case of a smooth algebraic group scheme. Our algebraic approach will be independent (even complementary) to the analytic approach taken in the course on Lie groups.

Second quarter: Algebraic Groups II, Julia Pevtsova

Upon developing tools for working with group schemes and Hopf algebras we will pursue the goal of classifying semisimple algebraic groups. Our approach will be based partially on the observation due to Weil that every adjoint simple algebraic group of classical type occurs as a connected component of an automorphism group of a central simple algebra with involution. This will complement the classical approach due to Borel.

Topics at a glance:
-Infinitesimal theory: differential and Lie algebras of affine group schemes;
-Borel fixed point theorem, parabolic subgroups, flag varieties;
-Structure theory for split semisimple algebraic groups: maximal tori, root systems, Weyl groups and Dynkin diagrams;
-Root datum and classification theorem;
-Representations: Highest weight theory (if time allows).

References:
(1) Introduction to Affine Group Schemes, W. Waterhouse;
(2) Representations of Algebraic Groups, J. Jantzen;
Prerequisites. Modern Algebra 504/5/6. The course will be suitable for a 2nd year or above graduate student leaning towards an algebra-related field (understood broadly: combinatorics, representation theory, algebraic geometry, algebraic topology). The second year graduate algebra course “Algebraic structures” is desirable but not required.

582 C: Pal - Optimal Transport II

The modern theory of Monge-Kantorovich optimal transport is barely three decades old. Already it has established itself as one of the most happening areas in mathematics. It lies at the intersection of analysis, geometry, and probability with numerous applications to physics, economics, and serious machine learning.

This year-long graduate topics course will serve as an introduction to this rich and useful theory.

The textbooks will be [1] and [2]. This will be supplemented often with papers, especially on the very recent approach by the so-called Schrödinger problem. We will roughly follow the following outline.


Spring: Schrödinger problem. Particle systems and the probabilistic view of optimal transportation. Gradient flow of entropy for Fokker-Planck equations.

References


Prerequisites: Measure theory and some topology and functional analysis. Basically the graduate real analysis sequence. It is useful to have some knowledge of probability (large deviations, Brownian motion) and Riemannian manifolds, but not necessary, since we will develop some of these notions on the way.

582 E: Viray - Rational Points I

Fix a field $k$ and an algebraic variety $X/k$. This course is concerned with the fundamental question: what can we say about the $k$-rational points on $X$. We will give an overview of this question in general, but the bulk of the class will focus on fields of arithmetic interest, namely global fields, local fields, and finite fields. Special topics include: Galois descent, the Brauer group, the Weil conjectures, cohomological obstructions, and arithmetic of rational surfaces.

Prerequisites: 504/5/6, familiarity with basic algebraic number theory and algebraic geometry.
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This year-long graduate topics course will serve as an introduction to this rich and useful theory.

The textbooks will be [1] and [2]. This will be supplemented often with papers, especially on the very recent approach by the so-called Schrodinger problem. We will roughly follow the following outline.


Spring: Schrodinger problem. Particle systems and the probabilistic view of optimal transportation. Gradient flow of entropy for Fokker-Planck equations.

References


Prerequisites: Measure theory and some topology and functional analysis. Basically the graduate real analysis sequence. It is useful to have some knowledge of probability (large deviations, Brownian motion) and Riemannian manifolds, but not necessary, since we will develop some of these notions on the way.

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