MATH 583E

MW 9:00 - 10:20

Mathematical General Relativity

Spring, 2017

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Course description. Mathematical general relativity is a well-established and vibrant branch of mathematics. It ties fundamental problems of gravitational physics together with beautiful questions in mathematics. The focus is on the study of manifolds equipped with a Lorentzian metric satisfying the Einstein field equations, and the tools used involve differential geometry, partial differential equations, conformal geometry and dynamical systems. There are many unresolved mathematical problems of great physical signiffcance in general relativity. The aim of this course is expose students working in a wide variety of fields to the mathematics underlying this fundamental scientific theory (which describes to stunning accuracy the large scale structure of our Universe) and to provide a foundation for the further study of mathematical general relativity.

Outline: Some of the main topics which may be covered are as follows.

- Introductory Material
  - Minkowski space Special Relativity
  - Stress-energy tensors and classical fields in spacetime
  - The Einstein field equations
  - Energy conditions
- Casual Structure
  - Global hyperbolicity and Cauchy developments
  - Domains of dependence, Asymptotic structure and causal boundaries
- Initial Value Problem
  - Initial data sets: The Einstein constraint equations
  - Local well-posedness results
- Black holes
  - Event horizons
  - Singularity theorems

- Possible Additional Topics
  - Positive mass theorems
  - The topology of higher dimensional black holes
  - Penrose inequalities
  - Quasi-local mass
  - Gravitational waves
  - The "standard model" for cosmology

Prerequisites. We will assume a basic knowledge of Riemannian Geometry as is usually covered in Math 547 or in "Riemannian Manifolds: An Introduction to Curvature" by John M. Lee. Results on the local well-posedness of the Einstein equations depend on local existence and uniqueness results for quasi-linear hyperbolic PDE. The extent to which these will be explored in detail will be dictated, in part, by the interests of the students.