Math 582F, Winter 2017

MWF 11:30 - 12:20

Pseudodifferential Operators and Calderón-Zygmund Theory

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Course description. This course will be an introduction to the fundamentals of pseudodifferential operators, and to basic results in the theory of Calderón-Zygmund operators. Both subjects are of importance to a range of fields in analysis, including probability, partial differential equations, and geometric measure theory. Pseudodifferential operators provide a convenient way to represent the fundamental solution (Green's kernel) for variable coefficient elliptic partial differential equations. They are also the core tool in microlocal analysis, which concerns itself with a finer understanding of the singularities of distribution solutions to partial differential equations. We will cover all basic details of the calculus for the standard class of pseudodifferential operators on $\mathbb{R}^n$, including composition and global L^2 boundedness results, and as an application, the definition and properties of wavefront sets.

Calderón-Zygmund theory is the branch of harmonic analysis focused on extending theorems and tools of Fourier analysis to the setting of L^p spaces, for $1 < p < \infty$, where the Fourier transform is no longer a unitary map. More deeply, it concerns understanding in greater detail how operations that may be expressed in terms of the Fourier transform (such as pseudodifferential operators) act on functions, by representing them as singular integral kernels. This understanding leads to interesting new results for L^2 functions as well.

Topics we will cover include:

- Symbol classes and oscillatory integrals
- The composition calculus for pseudodifferential operators
- Boundedness on $L^2(\mathbb{R}^n)$ and on Sobolev spaces
- Microlocalization and wavefront sets
- The maximal function and the Calderón-Zygmund decomposition
- Calderón-Zygmund convolution kernels and L^p boundedness
- Pseudodifferential operators as an example of Calderón-Zygmund operators
- L^p regularity for variable coefficient elliptic PDE's
- Sobolev spaces based on $L^2(\mathbb{R}^n)$

Prerequisites. Knowledge of Lebesgue measure on Euclidean space, and of the Fourier transform at the level of Math 526/556. Also, Math 557 or the permission of the instructor.

Text:

1. The text of Xavier St. Raymond, *Elementary Introduction to the Theory of Pseudodifferential Operators* is the best reference for the first part of the course. The material on Calderón-Zygmund operators can be found in multiple references, some of which will be held on course reserve, but our presentation will be self-contained.