Math 327 Syllabus

Textbooks: *Advanced Calculus* (Second Edition) by Patrick M. Fitzpatrick and *Principles of Mathematical Analysis* (Third Edition) by Walter Rudin. Fitzpatrick is the main text. In the following, references to Rudin' book are indicated by [R]; references without this indication are to Fitzpatrick's book.

Note: Fitzpatrick delays the use of epsilons and deltas in his treatment of continuity, using only the sequence definition for most of the chapter. In the case of uniform continuity he avoids the ϵ-δ definition entirely. The following contains guidelines aimed at rectifying this bias.

1. The Real Number System (Ch. 1 & [R] p. 1-10.) [5 - 6 lectures]
Quick review of sets, functions, equivalence relations, $\mathbb{N}, \mathbb{Z}$ and $\mathbb{Q}$
Fields, ordered fields, and their properties
There is no rational number x such that $x^2 = 2$. (Usual classical proof.)
The least upper bound property, $\mathbb{R}$, suprema and infima
The Archimedean property
Intervals, absolute value and the triangle inequality
Density of $\mathbb{Q}$ in $\mathbb{R}$
For $c > 0$ and $n \in \mathbb{N}$, there exists a unique $x \in \mathbb{R}$ such that $x^n = c$. (Prove $x = \sup\{r : r^n < c\}$.)

2. Sequences (§2.1 - §2.4, part of §9.1, [R] p. 56-58.) [8 lectures]
Sequences and limits: basic definitions and facts (§2.1, and Theorems (2.18) and (2.19))
Sandwich (squeeze) theorem
Standard sequences: $\frac{1}{n^p}, c^{\frac{1}{n}}, n^{\frac{1}{n}}$ ([R] Theorem (3.20) (a) - (c))
Monotone sequences (§2.3)
Subsequences and sequential compactness (§2.4)
$\liminf$ and $\limsup$: define, and prove $\liminf = \limsup$ (as a finite number) iff limit exists
Cauchy sequences, and completeness of $\mathbb{R}$ (§9.1 through Theorem (9.4))

3. Continuous Functions (§3.5, §3.1 - §3.4, §3.6) [6 lectures]
ϵ-δ definition of continuity, characterization using sequences, algebra, compositions (§3.5 and §3.1)
Extreme Value Theorem (§3.2)
Intermediate Value Theorem (§3.3)
Monotone functions: continuity and inverses (§3.6)
Uniform Continuity: ϵ - δ definition of uniform continuity, proof that the definition given in §3.4 is equivalent to the ϵ - δ definition, continuous $f : [a, b] \to \mathbb{R}$ is uniformly continuous (§3.4)

4. Series (§9.1 & parts of [R] Ch. 3) [5 - 6 lectures]
Convergence and Cauchy criterion for series, absolute convergence
Series of non-negative terms: comparison test, Cauchy Condensation Theorem ([R] Theorem (3.27))
Standard series: $\sum x^n, \sum \frac{1}{n}, \sum \frac{1}{n^p}$ (Do not cover the integral test, use CCT instead.)
Root test and ratio test ([R] p. 65-69)
Alternating series, $\sum (-1)^n \frac{1}{n}$, re-arrangements ([R] Example (3.53))
Re-arrangements of absolutely convergent series ([R] Theorem (3.55))
(Time permitting: Cauchy product of series ([R] Theorem (3.50))